PEACE WITH ZENO

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Suppose a particle traverses some given distance in a certain period of time ‘t’. Its movement is constituted by its being in one place at one time and in an other place at another time as well as the time ‘t’ is taken up by its transitions or by its being in different places at different times.

To facilitate the work of imagination, let us take into consideration only the fore-end of the particle. The movement of the particle involves the disappearance of its fore-end from its initial position and appearance of it at some other position. It can be asked, “Does the fore-end vacate its initial position at the beginning of the interval ‘t’’ or does it not?” If it does not, it is still at its initial position and the movement does not commence at the beginning of the interval ‘t’ and thus, not the interval ‘t’ but some other interval is correlated with the motion, which is contrary to our supposition. Therefore, the fore-end must leave its initial position at the beginning of the interval ‘t’. Now, if it leaves its initial position at the beginning of the interval ‘t’, it can be asked, “When does it land into some other position?” If it does not land into anyother position for any interval of time, it must be no where or in a state of non-existence, because it is neither at its initial position nor at anyother position. This will mean a void in the life history of the particle and will be ruinous to the continuity of its existence. It must, therefore, land into some other position, not subsequently, but at the very beginning of the interval ‘t’. This means that its acts of leaving the initial position and landing into another position do not require any period of time. Now the moving end will either stay at this position for some period or it will not stay there for any period. If it does not stay there for any period, but leaves it and lands into yet another position, this must be done at the very beginning of the interval ‘t’, because by the conclusion just established, the acts of leaving one position and landing into another do not require any period of time. The state of affairs for all other positions will be the same and the correlation of a period of time with movement will be absolutely excluded which will be contrary to our supposition. The fore-end must, therefore, stay for a small period of time at every position and it will be this stay which will require a period of time. The awkward phrases ‘the act of leaving’ and ‘the act of landing' can now be replaced by the single phrase, 'the act of transition' from which they are deriveable. The conclusion is that the period of time is required for the various 'states of rest' of the fore-end at different positions and not by the acts of transitions.

A period of time, say an hour, is enclosed within two terminals or limits, its beginning and end. Its first limit which marks its beginning also marks the end of the previous hour, Likewise, its second limit which marks its end also marks the beginning of the subsequent hour. Its two limits are, thus, shared by it with the previous and the following hours. Any portion of this hour, a minute or a second is enclosed within two ends or limits. Note that the limits or ends are not time and have no actuality apart from that which lies in-between and that which lies in- between is a period or a portion of time and is termed an hour or a minute or a second according to its magnitude. If we define an instant as the limit, i.e. the beginning or end of some period of time, we can say that the transition of the moving particle from one position to another is accomplished in an instant and does not require a period of time for its execution. What requires a period of time is the state of immobility of the particle. The movement of the particle is constituted not merely by its instantaneous transitions but also by its states of rest for some period of time at every position.

With this account of motion, we can confront the paradoxes of Zeno. The first paradox[59] called Dichotomy is as under:

'You cannot get to the end of the race-course. You cannot traverse an infinite number of points in a finite time. You must traverse the half of any given distance before you traverse the whole, and the half of that again before you traverse it. This goes on ad infinitum so that there are infinite number of points in a given space, and you cannot touch an infinite number one by one in a finite time.'

Zeno is at fault to conceive of the indefinite divisibility of a given space but not to conceive of the indefinite divisibility of a finite time. Any way, we may overlook this and proceed straight to the proposed solution.

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[59] The text of the paradoxed is by Burnet as given in Our Knowledge of the External World by Bertrand Russell, George Allen & Unwin, London 1914, Reprint 1961, pages 176-180

To traverse the race-course, the fore-end of our particle must effect a transition from its initial position to some other position. If it does not do so, it remains where it was and no movement takes place. But if it effects a transition, there must be some distance between its initial position and the new position, otherwise it will still be where it was and there will be no traversing of the race-course. According to the conclusion established above, the transitions are effected in 'no time', i.e. instantaneously. Hence the fore-end must be considered to have taken a sudden jump from the initial position to the new position, without touching the intervening finite or infinite number of positions between these two. In order that a period of time may be correlated with traversing, the fore-end must, stay at the new position for a certain period of time and then take another jump to another position and so on to the end of the race-course.

The second paradox termed Achilles is as below:

"Achilles will never overtake the tortoise. He must first reach the place from which the tortoise started. By that time the tortoise will have got some way ahead. Achilles must then make up that, and again the tortoise will be ahead. He is always coming nearer, but he never makes upto it."

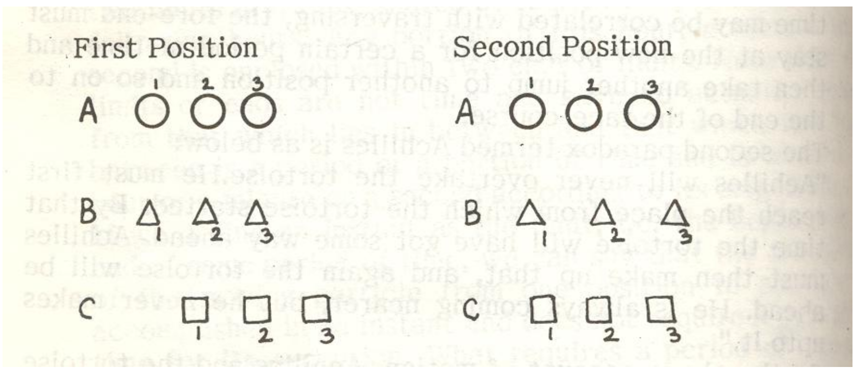
On the above account of motion, Achilles and the tortoise will be utilizing at their respective positions of rest different periods of time, those of Achilles being shorter. Achilles will, therefore, make upto it' and pass it at a certain position where the tortoise will be immobile for a longer period of time. On the doctrine of jumps Achilles will be executing longer period of time. On the doctrine of jumps Achilles will be executing longer jumps and in consequence, will be having fewer stations of halt.

The third paradox that of the arrow is as follows:-

"The arrow in flight is at rest. For, if everything is at rest when it occupies a space equal to itself, and what is in flight at any given moment always occupies a space equal to itself, it cannot move."

According to the viewpoint developed here, the arrow is certainly at rest at every moment, if the word moment stands for some period of time. Its movement, however does not lie in its states of rest alone, but also in its transitions which are effected instantaneously. The arrow in flight, therefore, effects instantaneous transitions also and thereby executes its movement.

The fourth paradox known as the Stadium is as under:-'Half the time may be equal to double the time. Let us suppose three rows of bodies one of which (A) is at rest while the other two (B,C) are moving with equal velocity in opposite directions. By the time they are all in the same part of the course, B will have passed twice as many of the bodies in C as in A. Therefore, the time which it takes to pass C is twice as long as the time it takes to pass A. But the time which B and C take to reach the position of A is the same. Therefore, double the time is equal to half.'



Zeno seems to assume that a body takes equal times to pass with the same velocity two similar bodies one of which is at rest and the other in motion. This assumption is wrong. But, be that as it may, on the hypothesis of instantaneous jumps, equal velocity will mean equal periods of rest and equal length of jumps. Therefore, if B and C as in the first position take their equal and instantaneous jumps at the end of the same period of time, they will all be found for the ensuing period in the same part of the course as in the second position and there will be no question of half the time being equal to double the time.

This paradox is interpreted by some[60] to be aimed at invalidating the assumption that a finite period of time consists of a finite number of

[60] Bertrand Russell Our Knowledge of the External World, Geroge Allen & Unwin, London, page 182.

moments. In the first position of the diagram B3 and Cl are opposite each other at the first moment. At the second moment in the second position B1 has come up opposite Cl. At what moment, then, did B2 and Cl pass each other? It must have been at some moment between the first and the second moments which, therefore cannot be consecutive, though they were supposed to be such. Accordingly, there must be other moments and an infinite number of them between any two given moments.

The word moment in the above interpretation, obviously stands for a period of time. According to the doctrine of motion and time developed here, a finite period of time will consist of a finite number of periods whereas according to the above interpretation, a finite period turns out to consist of an infinite number of periods. In this interpretation, however, continuous motion is being assumed, but in our doctrine motion cannot be continuous. By means of the instantaneous transitions in the opposite directions at the end of the first moment B1 and Cl can fall into line opposite each other for the second moment and in view of the motion of jump, the question, 'when did B2 and Cl pass each other' will not arise,

Our doctrine of motion may be stated as under:-

A particle which moves, takes a sudden jump from its initial position to a new position, stays there arrest for a small period of time, then takes another jump to another position and so on and so forth. The jumps are instantaneous, the points between the two positions of rest are not touched and the period of time correlate with motion is taken up by the periods of rest at every position. The concise manner in which this doctrine meets the difficulty of Zeno is its chief recommendation. The doctrine is conceivable and has suggested itself to human mind. Russell's version of it is the following[61]:

'All motion might consist of periods of rest separated by instants of infinite velocity.'

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ii G.J Whitrow in the Natural Philosophy of Time, Second addition, Clarendon Press, Oxford, 1980 page 191.

[61] Human knowledge: Its Scope and Limits by Bertrand Russell, George Allen and Unwin, London, Fourth Impression 1961, page 383.

An 'instant of infinite velocity' is no other than the 'instantaneous transition' of our phraseology. Unfortunately, the doctrine occurred to Russell's mind in connection with the matter of determination of velocity and not in the context of Zeno's paradoxes.

The doctrine has nothing of logic against it. In fact, by far the best manner, logically, of getting to the end of the race-course is to take a sudden jump from the initial position and land instantly at the end, without touching any point or wasting any time on the way. But as we do not go about our travels in this way, we must contemplate in our account a very large number or halts and shorter jumps short enough to present the perspective of a continuous movement.

The notion "jump" was conceived by Nazzam, a Muslim scholastic of early medieval period to meet a difficulty such as that in the Achilles paradox, It has been introduced into atomic physics by N. Bohr to account for the transitions in the atom of electrons from one stationary orbit to another. Bohr's insight, however, left unclarifled the manner of electron jump. It fell to the lot of Werner karl Heisenberg to supply the deficiency. It is said[62] that the physicists of Heisenberg's time were making every effort to find out what happens to the electron during its jump between the two orbits, when at some moment it struck him 'that the electron just never happens to be "between" the stationary' orbits. This intuition of Heisenberg led him to the formulation of the quantum, matrix mechanics.

At about the same time A.N. Whitehead[63] was assuming that the electron does not traverse its path in space continuously, but appears at discrete positions, remaining at each position for successive periods of time, like an automobile with an average speed of thirty miles an hour, which does not traverse the road continuously but appears successively at successive mile stones, remaining for two minutes at each mile stone'.

Heisenberg's intuition and Whitehead's assumption did not allude explicitly to the instantaneous nature of the jumps. The word jump is,

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[62] In Quest of the Quantum by L. Ponomarev, English Translation by Nicholas Weinstein, Mir Publishers Moscow 1973 Page 184.
[63] Science and the Modern World by A.N. Whitehead, A Mentor Book, 1957 pages 36-37.

however, misleading. The moving particle may be conceived to vanish, from one position and emerge at another at one and the same instant, like two light-bulbs, one of which may be switched off and the other switched on instantly with a single switch-button.

Space and time are ordinarily considered to be continuous. On our account of motion, they are atomised, time by the instantaneous transitions and space by jumps. Transitions or individual acts or occurrences which are synonymous in the present context, are instantaneous, i.e. indivisible and unextensive in time. It is these individual acts or occurrences which furnish a period of time with its limits, i.e. the beginning and end and thereby supply human mind with the notion 'instant'. Continuity of movement stands abolished in this account.

If all motion consists of periods of rest separated by instantaneous jumps, the fact that in daily life we observe many objects to be visible moving and do not observe the disjointed motion, requires to be accounted for.

The explanation of visible movements of daily life, such as that of a motor car on the road, is not far to seek. If the movement of an object is of a suitable speed, so that the object can be seen in more than one position in a single sensation and also in some of these positions earlier than in others, the object will be seen to be moving. This is due to the phenomenon termed 'persistence of vision', whereby the brain retains the impression of an object; for the fraction of a second longer than the time of its actual exposure before the eyes. It is owing to this phenomenon that stationary photographs on a film, when run sufficiently rapidly through a cinema projector, present the view of a moving object.

The non-observableness of the discontinuous, jerky motion is due to the fact that our senses are not adequately acute and precise. An ordinary object, a stone for example, appears to be continuous & of a single piece, whereas physics tells us that if is constituted by billions of tiny particles, with vast stretches of space between them. We do not see the spatial gaps between the particles nor even the particles themselves. In movement, the particles constituting an object, may not all jump at the same instant, the period of stay of the individual particles may be very brief and the length of the jumps may be too small to be discriminated. If the lengths of the jumps were considerable, the periods of stay appreciable and the jumps of the particles simultaneous, perhaps we would have experienced the discontinuous motion. According to Russell[64],

'A world in which all motion consisted of a series of small finite jerks would be empirically indistinguishable from one in which motion was continuous.'

Our account of motion as made up of immobilities, i.e. periods of rest with instantaneous transitions, abolishes the notion of the perceived 'state of motion'. There are not two states, the state of rest' and' the state of motion.' There is only one state,'the state of rest' which a body always takes. By the abandonment of the notion of 'the state of motion' we have met Zeno half-way. If he could agree on the notion of the instantaneous transitions, the debate with him would be over for good.

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64 Introduction to Mathematical Philosophy Bertrand Russell, George Allen & Un win, London 1919, Tenth Impression 1960, page 140.