TIME IN SPECIAL RELATIVITY THEORY

(Part-I)

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1. PROPAGATION OF LIGHT

The nervous probandi of the special relativity theory is the law of the propagation of light in vacuo. The time concept of the theory is entirely based on this law. The mere statement that the velocity of light is constant and its value is C in empty space is not very informative as to its implications and role in, the special theory of relativity. In this form, it does not disclose certain features of the law which are important for the theory and particularly for its time concept. These features are:-

I- The "velocity of a ray of light which may propagate in one inertial system is constant and its value is C in that system. The velocity of a second ray of light which may propagate in a second inertial system is also constant and its value is C in the second system. This feature calls attention to two separate rays of light propagating in two separate inertial systems and is implicit in the theory, but is seldom mentioned and never commented upon.

Some use of this feature will be made in the sequel.

II- The velocity of one and the same ray of light is to be treated as C as judged in each of the inertial systems which are moving with respect to one another. Einstein's authority for this is the following[80]:

- The relation [between the values x, y, z, t and x', y', z', t', of an event with respect to the inertial systems K and Ḱ] must be so chosen that the law of the transmission of light in vacuo is satisfied for one and the same ray of light (and of course for every ray) with respect to K and Ḱ

[80] A. Einstein, Relativity: The Special and General Theory, Methuen, London, page 32.

The paradoxical aspect of this feature of the law has been brought out by A.N. Whitehead in the following words[81]:

For example, consider two cars on the road, moving at ten and twenty miles an hour respectively, and being passed by another car at fifty miles an hour. The rapid car will pass one of the two cars at the relative velocity of forty miles per hour, and the other at the rate of thirty miles per hour. The allegation as to light is that, if we substituted a ray of light for the rapid car, the velocity of light along the roadway would be exactly the 'same as its velocity relatively to either of the two cars which it overtakes.

This is because the roadway and each car, in ' turn, can be considered to be at rest and the ray of light to be passing along in the rest system of each at its constant velocity C.

This feature of the law of propagation of light is well-known, but is rarely commented upon. It implies that all inertial systems are equivalent for the propagation of light in vacuo, so that observers in each system can consider the velocity of one and the same ray of light to be C in their own system.

This feature is basic to the theory.

III) The velocity of a ray of light which may be initiated in any one of the infinite number of inertial systems, is to be considered by observers of every inertial system to be C in their own system only and c-v or c+v in every other inertial system, keeping in view the direction of movement of the ray and of the other system. For example, suppose a ray is initiated and propagates in the inertial system K at the velocity C, then observers in every other inertial system K3,K4 etc, will consider this same ray to be propagating at the velocity C in their own system only, and in the system J, or any other inertial system to be propagating at the velocity c-v or c+v, keeping in view the direction in which the' other system might be moving, considering, of course, the Iength to be contracted in the direction of movement of the moving system. Einstein's authority for this is as under. While deriving the

[81] A.N. Whitehead, Science and the Modern World, Mentor Books, page 119.

Lorentz transformation in his first paper on special theory of relativity, he considers two inertial systems K and K'where K'is moving and then writes the following[82]:

From the origin of the system K let a ray be emitted at the time to along the x-axis to x, and at the time t, he reflected thence to the origin of the coordinates, arriving there at the time t: we then must have $\frac{1}{2}\left(t\frac{1}{0}+t\frac{1}{2}\right)+t'_1$, or, by inserting the argument of the function t and applying the principle of the constancy of the velocity of light in the stationary system;

$$\frac{1}{2}\left[(0,0,0t)+t'\left(0,0,0,t+\frac{x'}{c-v}\right)\right]$$

The ray is emitted from the origin of the moving system K' and from the point of view of observers in the stationary system K, it propagates in their own system at the velocity C, but advances towards) X' (which is fixed'in K' at the velocity c-v, because light is advancing at the velocity C and X is moving away in the forward direction at the velocity v, [so light overtakes it at the velocity c-v]. On return journey, as judged from the system K, the ray of light advances towards the origin of K at the velocity C and the origin of K' comes forward to meet it at the velocity V. Therefore, on return journey the ray of light approaches the origin of K' at the velocity c + v.

A little further on Einstein writes as below[83]:

But the ray moves relative to the initial point of K', when measured in the stationary system, with the velocity c-v, so that $\frac{x'}{c-v}=t$.

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[82] A. Einstein, "Electrodynamics" in the Principle of Relativity, Dover Publications Inc. page 44.

[83] Ihid, page 45.

Very few admirers of the special theory of relativity seem to be aware of the fact that in this theory, the -velocity of light is treated in the moving inertial systems, from the standpoint of the stationary systems, as c-v or c+ v, keeping in view the direction of movement of the ray of light as well as that of the moving inertial system. This fact is never commented upon, but is quietly acquisced in even by the those who have noticed its[84].

According to the view to be developed in the present discussion, this factor exemplifying c-v or c+v is the major culprit by which the simple-minded time concept of the ordinary mortals is subverted to the astounding consequences in Einstein's special theory of relativity. One of the most important results of the theory is that "a moving clock runs slow by the factor $\sqrt{1-\frac{v^2}{c^2}}$ "or alternately "time itself slows down in the moving systems by this factor". Thus, if a cosmonaut goes off on space travels at thigh speed, on his return to earth, he will be found to have aged less and will be younger than his twin brother whom he left on earth. This is termed as the "twin paradox" or the "clock paradox" and 'is considered as a means of "perennial youth" for the cosmonauts.

The contention in this paper is that it is not time or the clocks which slow down, but it is the manner in which time is measured (or rather calculated) by means of one and the same ray of light which is considered to be propagating at the velocity c-v or c+v in the moving systems, but at the velocity C in the systems considered to be stationary.

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2. THE IDEAL EINSTEIN-LANGEVIN LIGHT CLOCK

[84] Clement V. Durell, Read-able Relativity G. Bell and Sons, London 1926.

For the purposes of time measurement, apart from ordinary clocks, relativity literature contemplates a light clock which is termed the ideal[85] Einstein-Langevin Clock. Einstein's version of this clock is as under[86]:

....a light signal, which is reflected back and forth between the ends of a rigid rod, constitutes an ideal clock, provided that the postulate of the constancy of the light-velocity in vacuum does not lead to contradiction.

Edwin F. Taylor and John Archibald Wheeler write in their book,

Space-Time Physics [87]:

when a mirror is mounted at each end of a stick one-half metre long, a flash of light may be bounced back and forth between these mirrors; such a device is a clock.

Einstein wrote in his book, Meaning of Relativity[88] .

....it should be noted that a light signal going to and fro between S1 and S2 would constitute a clock.

Here S1 and S2 are two stars and the distance between them is very great. Neither Einstein nor any other relativist imposed any restriction on the length of the rod which may be of any length.

3. RATES OF TIME LAPSE IN THE SYSTEMS K AND K' .

We are in a position now to calculate time by means of the Einstein-Langevin light clock.

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85 L. Marder, Time and Space Traveller, University of Pennsylvania Press, page 40.
86 P.A. Schilpp (ed.) "Autobiographical Notes" in Albert Einstein, Philosopher-Scientist, quoted in Problems of Space and Time by J. J. C. Smart, Problems of Philosophy Series, page 281.
87 Edwin F. Taylor and John Archibald Wheeler, Space Time Physics page 4.
88 Albert Einstein, Meaning of Relativity, Methuen & Co Ltd. page 122.

Let us imagine two inertial systems K and K' in uniform, relative motion along the exis of x. Their respective origins O and O' coincide at zero hour which is the same instant in both the systems and from which time is to be calculated. Their axes are parallel and their supposed relative velocity v is 4 legs per second, the velocity of light being 5 legs per second. On this scale, one leg measures a distance of 37200 miles, so that 5 legs will be equal to 18600.0 miles. A rigid rod AB, 5 legs long with reflecting mirrors at each end is placed along the exis of x in the system K' and the end A of the rod coincides with the origin O'. A light ray is emitted at zero hour from the end A and travels to the other end B where it is immediately reflected back towards A. This constitutes the Einstein-Langevin ideal clock and with its help, we calculate the rate of time lapse in the system K' as compared to the rate of time lapse in the stationary system K.

In the ensuing conceptual explorations which in relativity literature are given the respectable name of "thought experiments", the result of length contraction by the factor $\sqrt{1-\frac{v^2}{c^2}}$ of the special theory of relativity in the moving systems will be assumed throughout.

As the rod AB is 5 legs in the system K' and as the velocity of light will be treated as C in this system, time at the end B will be one K'-second when the ray reaches this end. After reflection at the end B, it will reach the end A in another K' second and time at this end will be 2 K'-seconds on arrival back of the ray.

The propagation of this ray of light as judged from the system K, will be as under:

The system K' with the rod AB in it, will be judged to be moving towards the right i.e. towards the positive side of the x axis at 4 K-legs per one K-second and the length of the moving rod AB, will be considered to be shortened on account of its movement relative to the system K by the factor $\sqrt{1-\frac{v^2}{c^2}}$ or $\sqrt{1-16/25}$ or $\sqrt{16/25}$ or $3/5$ in our numerical example, and will be equal to $\lfloor 5x \, ^3\!/_5 \rfloor$ 3 K legs. The principle of the constancy of the velocity C of light will now be applied in the system K and the same ray of light will advance in it at 5 K-legs per one K:second and the rod AB will move forward at 4 K-legs per one K-second. The velocity of the ray on the rod as judged from the system K will, therefore, be c-v and it will cover a distance of (5-4) one K-leg on the rod in one K-second. It has to cover a distance of 3 K-legs i.e. 5 K'-legs shortened, which will be convered in 3 K-seconds. Thus, when the ray of light arrives at the end B, time at this end will be one K -second and opposite B in the system K, it will be 3 K-seconds. In 3 K-seconds, the ray of light will have covered a distance of [5 x3] 15 K-legs and will have arrived at leg 15 of the system K.

So the coordinates in the system K of the event of arrival of the ray at leg 15 thereof will be,

X = 15, t = 3

From these, the coordinates of the same event in the system K'hy means of the Lorentz transformation:

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}, t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$X' = 5 \rightarrow \frac{3}{5}[15 - 4 \times 3] or \frac{5}{3}[15 - 12] or \frac{5}{3} \times 3 = 5$$

$$t' = 1 \rightarrow \frac{3}{5}\left[3 - \frac{4}{25} \times 15\right] or \frac{5}{3}\left[3 - \frac{12}{5}\right] or \frac{5}{3} \times \frac{3}{5} = 1$$

Thus our calculations of one K'-second at the end B at a distance of

5 K'-legs from the end A and 3-K seconds at leg 15 of the system K opposite the end B, are in accord with the Lorentz transformation.

It may be noted that the ray of light travels a distance of 5 K'-egs in one K'-second in the systems K' and a distance of 15 K-legs in 3K - seconds in the system K'. The distance and time values in the system K are three times the values in the system K'. [This fact will help

simplification of the calculations in the sequel.] The excess values in the system K are obviously due to the fact that the velocity of light is treated as C in the system K, but as judged from this system, it is treated as c-v in the system the K'. On account of the movement of the system K with respect to the system K', the ray of light has to travel extra distance in the system K in order to catch up with the end B of the rod which is moving forward.

On reflection at the end B, the ray of light takes another one K' second to arrive back at the end A of the rod, so time at the end A is now 2 K'-seconds.

The return journey of the ray of light as judged from the system K, will be as under:

The distance AB will be judged to be shortened by the factor $\sqrt{1-v^2/c^2}$ or $3/5$ of our example and will be $\lfloor 5\times 3/5 \rfloor$ 3 K-legs as before. The ray of light will advance towards the end A at 5 K-legs pet-one K-second and the end A will move towards the on-coming ray of light at 4 K-legs per one K-second. The velocity of the ray of light on the rod AB, as judged from the system K will be C + V or [5 + 4] 9 K-legs per one K-second. So it will cover the distance of one K-leg in 1/9 K-second. It has to cover a distance of 3 K-legs which will be covered in [1/9 x 3] K-second. In 1/3 K-second the ray of light will cover a distance of [5 x 1/3] 5/3 K-legs and arrive back at [15 - 5/3] leg 40/3 of the system K.

Thus, when the ray arrives back at the end A of the rod, time in the system K will be $\lfloor 3+ 1/3 \rfloor 10/3$ K-seconds and in the system K' it will be [1+1} 2 K' seconds. Taking into consideration the outward and the return journey of the ray of light for equal distances both ways in the 1 system K, the rate of time lapse in this system will, therefore, be $\lfloor 2 \times 3/10 \rfloor or\ 3/5$ of the rate of time lapse in the system K. In other words, the rate of time lapse in the system K will be slow by the factor 3/5 which is in accord with the Lorentz factor

$$\sqrt{1-\frac{v^2}{c^2}} \text{ or } \sqrt{1-16/25} \text{ or } \sqrt{9/5} \text{ or } 3/5$$

In $10/3$ K-seconds, the end A of the rod will have reached $\lfloor 4 \times 10/3 \rfloor$ by $40/3$ of the system K to receive back the ray of light at this juncture. So the coordinates in the system K of the event of arrival of the ray of light at the end A opposite leg $40/3$ of the system K will be

$$x\, 40/3, t = 10/3$$

From these by means of the Lorentz transformation,

$$x' = o\frac{3}{5}\left[\frac{40}{3} - 4 \times \frac{10}{3}\right] or \frac{5}{3}\left[\frac{40}{3}\frac{40}{3}\right] or \frac{5}{3} \times 0 = 0$$

$$t' = 2\frac{5}{3}\left[\frac{10}{3} - \frac{4}{25} \times \frac{40}{3}\right] or \frac{5}{3}\left[\frac{10}{3} - \frac{32}{15}\right] or \frac{5}{3}\left[\frac{50-32}{15} or \frac{5}{3} \times \frac{18}{15} = 2\right]$$

our calculation of 2 K'-seconds at the end A and $10/5$ K-seconds opposite the end A at leg $40/3$ of the system K are, therefore, in accord with the Lorentz transformation.

It may be noted that the ray of light now travelled back a distance of 5 K'-legs in one K'-second, but only a distance of $\left[5\times 1/3\right]$ $5/3$-legs in 1/3 K-second. The distance and time values in the system K are now only one third of the distance and time values of the system K' [This fact will be employed for the simplification of our calculations in the sequel.] The small values now in the systern K are due to the fact that the velocity of light is treated as C in the system K, but as judged from this system, it is treated as c+v [5+4] 9 K-legs per one K-second in the system K'.

The total time of travel of the ray of light from the end A to the end B and back is [1 + 1} 2 K'-seconds and $\left\{3+1/3\right\}10/3$ K'-seconds. In the system K' or on the Einstein-Langevin clock the ray of light took equal times for its outward and return journey, but in the system K, it took nine times more time for its outward travel and only one—tenth of the total time for its return travel. In the system K' it travelled equal distance for its two ways travel, but in the system K, it travelled 15 K-legs for its outward journey and only $5/3$ K-legs for its return journey, thus nine times more distance for its outward travel.

OBSERVATIONS

a). As the velocity of the ray of light has been treated as C in the system K, time may be said to have run uniformly in this system, both for the outward as well as backward travel of the ray, whereas in the system K', from the standpoint of the system K, time will be considered to have run non-uniformly. So, if an ordinary clock was placed at the end A of the rod, the relativity physics will demand of it to record its first one second for 3 of the K-seconds, that is, to tick three times slower than the K-clocks and to record the next one second for $\frac{1}{3}$ of the K-second, that is, to tick now three times faster than the K-clocks. And if a cosmonant was sitting beside it, his heart-beat, his breathing, his blood circulation and all his other physiological processes will slow down three times for the first 3 K-seconds and will quicken up three times for the next $\frac{1}{3}$ K-second, thus losing altogether their gear and rhythm.

It is doubtful whether the ordinary clocks can oblige special relativity theory to behave in such and anomalous manner and is much more incredible whether biological clocks can behave so. Accordingly, "perennial youth" need not be dreamed of by the cosmonauts.

(b) We have arbitrarily supposed the length of the rod to be 5 K'-legs. It can be supposed to be of any length. Moreover, two lengths of equal magnitudes can be supposed, one lying parallel to the x axis and the other lying perpendicular to it, like two arms of the Michelson-Morley interferometer. Time lapes on the length lying parallel to the direction of movement will be irregular and non-uniform, whereas on the length lying perpendicular to the direction of movement, it will be regular and uniform. Relativists usually suppose the length to be lying perpendicular to the direction of movement for the purpose of illustrating the retardation of time in the moving systems. But nobody has given any cogent reason for so placing the rod and for not placing it in the direction of movement.

The rate of $\frac{3}{5}K'$-second. on the Einstein- Langevin clock for one K'-second which is in accord with the Lorentz transformation is not factual, because in actual fact, the rate is one K'-second for K-seconds for the first half of the K'-time one K'-second for 1/3 K-second for the next half of K -time. The rate of 3/5K -second for one K-second is derived by dividing [1+1] 2 seconds of the system K' by $\left[3\times\frac{1}{3}\right]\frac{10}{3}$ seconds of the system K and is, therefore, the arithmetical average rate and as such is artificial and not actual. In his first paper on special theory of relativity, Einstein imagined the ray of light to travel equal distances to a point x' and back in the system K' and then set up the following equation[89]:

$$\frac{1}{2}\left[t'(o,o,o,t)+t'o,o,o,t+\frac{x'}{c+v}+\frac{x'}{c+v}\quad)\right]=t\left(x',o,o,t+\frac{x'}{c-v}\right)$$

to derive the Lorentz transformation by further mathematical manipulation. The velocity of the ray of light as c-v in the moving system in one direction and c+v in the opposite direction is, therefore, implicit in the Lorentz transformation and does not seen to be gainsaid by deriving the transformation in some other way. In relativity literature, it is stipulated (albeit arbitrarily) that the unit of time on the Einstein-Langevin light clock should be taken as that quantity of time for which the ray of light returns to one end of the rod after reflection at the other end. But there is no satisfactory reason to take into account as a unit of time the to and fro journey as a whole and to pretend ignorance about the behaviour of time in the separate outward and inward travels of the ray of light when it is definitely known that for the outward travel of the ray, time in the system K' is thrice slower and for the inward travel, it is thrice faster than in the system K. In justification for taking the unit of time, the to and fro journey as a whole, the following reasons may be advanced, viz.

i- We cannot directly know the time at the other end of the rod which is far removed from us (in our numerical example 186000 miles away) or that,

ii- The velocity C of the ray of light for its outward and inward travel is only an average velocity.

The first reason is unacceptable in view of the fact that the distance between A and B can be made as small as we wish, so that the entire. rod may be visible to the naked eye at a single glance. Even then, according to our numerical way of calculations, half of the total time in the system K' will be three times slower and the other half three times faster than the respective times in the system K.

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[89] A. Einstein, "Electrodynamics" in Principle of Relativity, Dover Publications Inc. page 44.

The second reason is unacceptable for the simple fact that the constancy of the velocity of light means that it propagates in the isotopic, empty space at the same uniform, constant velocity C throughout its journey and also for the additional fact that in his first paper on special theory of relativity. Einstein established, "by definition that the "time" required by light to travel from A to B equals the "time" it requires to travel from B to A"[90] So there is no question of average velocity for the two way travels of the ray of light, in consequence of which an average of two times may be accepted as a unit of time. Accordingly, if time is to run in the moving; systems in conformity with the way in which light is judged from the stationary systems to be propagating in those systems at the velocity c- v in one direction and c+v in the opposite direction, the Lorentz factor for time retardation turns out to be specious and artificial, because it implies as in our numerical example, the average of, three times more time in the system Ḱ than one half of the total time of die Einsein-Lahgevin clock in the system Ḱ, and three times less time of the system K than the remaining half of the total time of that clock in the system K. This factor which seems to embody an average quantity of time retardation is, therefore, merely mathematical and as such grossly misleading.

C- When the ray of light reaches the end B of the rod opposite leg 15 of the system K,, time of the system K' is one second and of the system K' it is 3 seconds. In 3 K-seconds, the end A of the rod arrives opposite leg 12 of the system K at the velocity of 4 K-legs per one K- second. But the other system K' can also be considered to be at rest and the system K, to be in motion at the velocity of 4 K'-legs per one K'-second towards the negative side of the x axis. Length in the system K will be shortened on account of its motion and 4 K'-legs will measure the same distance as $\lfloor 4\times {}^{5}\!/_{3} \rfloor {}^{20}\!/_{3}$ - K-legs. Thus, in one K'-second, a distance of 4 K'-legs or 20/3 K-legs will pass in front of the end A and so it will be opposite leg 20/3 of the system K. Therefore, when the ray of light reaches the end B of the rod, opposite leg 15 of the system K', the end A of the rod is opposite leg 12 of the system K according to the standpoint of this system. But according to the standpoint of the system K', it .is opposite leg. of the system K when the ray of light reaches the end B of the rod opposite leg 15 of the system K. So, when the

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[90] /bid, page 40

ray of light reaches the end B of the rod, the end A of the rod is opposite two places in the system K, opposite leg 12 of this system and opposite leg 20/3 of this system.

This paradoxical result which arises from the standpoint of the two systems, seems to have missed so far the notice of the admirers of the theory and is apparently irreconcilable and irresolvable, even if it is said that the end A is opposite leg "on the system K at the time of one K second and opposite leg 20/3 of that system at the later time of 9/5 K'-seconds, because 1/5 K'-seconds will not be acceptable from the standpoint of the system K' to be the Ḱ time at the end A when the ray of light reaches the end B of the rod.

But when the ray of light returns to the end A, time in the system K is 10/5 seconds and in the system Ḱ, it is 2 seconds. In 10/3 K-seconds the end A of the rod moved to $\left[4\times{}^{10}\!/_{3}\right]\frac{40}{5}$ K-legs and in 2 K'-seconds, a distance of [4 x 2] 8 K'-legs or $\left[8\times{}^{5}\!/_{3}\right]\frac{40}{3}$ K-legs passed in front of the end A of the rod. Thus, the end A is now not opposite two places of the system K, opposite only a single place, leg ${}^{40}\!/_{3}$ of that system.

How has this happened?

On the return journey from the end B to the end A of the rod, the ray of light took 1/3 second of the system K and one second of the system K. on 1/3 K-second, the end A of the rod advanced a small distance of [4 x 1/3]4/3 K-legs from leg 12 of the system K and reached [4/3+ 12] leg of this system. On the other hand for one K'-second, a distance equal to another 4 K' legs or $\left[4\times{}^{5}\!/_{3}\right]\frac{20}{3}$ K-legs passed in front of the end A and so $\left[{}^{20}\!/_{3}+{}^{20}\!/_{3}\right]leg\frac{40}{3}$ leg of the system K arrived opposite the end A.

Einstein worked, perhaps unconsciously with two sorts of criteria for calculating the light travel time. In one system the ray of light is obliged to travel equal distances, out from and back to the point of emission of the ray. on the other system, this condition cannot be fulfilled by the same ray of light on account of its velocity as c-v or c+v in the moving system, and so it has to travel unequal distances out and back in this second system. Therefore, it takes proportionally more time to cover the longer distance for one of the two sides of its journey in the second system. No reason has ever been given to justify the use of such two sorts of criteria and so the criteria in question possess highly arbitrary looks.

4- A STATIONARY LIGHT CLOCK ALSO RUNS SLOW.

(a) It is one of the widely accepted results of the special theory of relativity that "a moving clock runs slow" or equivalently, "time itself slows down in a moving system." We endeavour to show below that even a stationary Einstein-Langevin clock will show less time than the time measured by the ray of light in a moving system in which such a clock is not used.

The system K' cari be supposed to be at rest and the system K to be in motion at 4 K'-legs per one K'-second towards the left, i.e. towards the negative side of the x axis. The Einstein-Langevin clock of rod AB, 5 K-legs long, is placed in the system K' as before and light travel time on this clock is to be compared with the light travel time of the same ray of light in the system K.

The ray of light will take one K'-second to travel from the end A to the end B and one more K'-second to travel back from the end B to the end A. Sot it will take 2, K'-seconds for its two way travel on the rod AB.

The velocity of the ray of light will be treated as C in the system K' and as judged from this system, the ray of light will advance towards the right at 5 K' -legs per one K' -second and the system K will move towards the left, i.e. towards the on-coming ray of light at 4 K'-legs per one K'-second. The velocity of the ray in the system K will, thus, be c+v and it will, therefore, cover a distance of [5 + 4] 9 K'-legs in the system K in one K'-second. But length in the system K will be contracted by the factor $\sqrt{1-{v^2}/{c^2}}$ or .3/5 of our example, due to its motion and 9 K'-legs will measure the same distance as [9 x 5/3] 15 K-legs. Thus after one K'-second when the ray reaches the end B of the rod, the leg 15 of the system K will have arrived opposite the end B. Time at leg 15 of the system K will be [15/3] 3 K-seconds.

The coordinates in the system K' of the event of arrival of the ray at the end B and opposite leg 15 of the system K will be

$$x' = 5, t' = 1$$

As the velocity of the system K towards the negative side of the x axis can be written as -v, the Lorentz transformation will take the from as

$$x = \frac{x' - vt}{\sqrt{1-\frac{v^2}{c^2}}} \text{ and } \qquad t = \frac{t' \frac{vx'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

But as two minus signs together make up one plus sign in algebra, the above equation can be written as

$$x = \frac{x' - vt'}{\sqrt{1-\frac{v^2}{c^2}}} \qquad t = \frac{t' \frac{vx'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

These equations are called the inverse Lorentz transformation. By the use of these, we will have

$$x = 15\, {5}/{3}\, [5 + 4 \times 1] or\, {5}/{3} \times 9 = 15$$

$$t = 3\tfrac{5}{3}\left[1 + \tfrac{4}{25} \times 5\right] or\ \tfrac{5}{3}\left[1 + \tfrac{4}{15}\right] or\ \tfrac{5}{3} \times \tfrac{9}{5} = 3$$

Thus our calculations of 3 k-seconds at leg 15 of the system K opposite the end B of the rod in the system K' and one K'-second at leg 5 of the system K' i.e. at the end B of the rod, are in accord with the inverse Lorentz transformation.

On its return journey from the end B, the ray of light will take one more K'-second to arrive at the end A and time at this end will be [1 + 1] 2 K'-seconds.

As judged from the system K', the ray of light will be heading towards the left at 5 K'-legs per one K'-second and the system K will be moving away towards the- left at 4-K'-legs per one K'-second. The velocity of the ray of light in the system K will, thus, be c-v and it will gain on the system K, a distance of [5-4] one K'-leg in one K'-second. As the length in the system K will be shortened by the factor 3/5 due to its movement, one K'-leg will measure the same distance as [1 x 5/3] 5/3 K-legs. To cover this distance of 5/3 K-legs, the ray of light will take $\left[\frac{5}{3} \times \frac{1}{5}\right] \tfrac{1}{3}$ K-second and will have arrived opposite [15 – 5/3] leg 40/3 of the system K. So when the ray arrives back at the end A of the rod in the system K', time at the end A is [1+1] 2 K'-seconds and opposite the end A of the rod at leg 42 of the system K, it is [3 +1/3]10/3 K-seconds.

The coordinates in the system K' of the event of arrival of the ray of light back at the end A, i.e.. at the origin of the system K' and at leg 40/3 of the system K, opposite the end A of the rod will therefore, be

$$x' = o \quad t' = 2$$

From these by means of the inverse Lorentz transformation:

$$x=\frac{40}{3}\quad {}^{5}\!/_{3}\left[o+4x2\right]or\ {}^{5}\!/_{3}\times 8={}^{10}\!/_{3}$$

$$t={}^{10}\!/_{3}\quad {}^{5}\!/_{3}\left[2+{}^{4}\!/_{25}\times o\right]or\ {}^{5}\!/_{3}\times 2={}^{10}\!/_{3}$$

Thus our calculations of 10/3 K-deconds at leg 40/3 of the system K, opposite the end A of the rod and 2 K'-seconds at the end A of the Einstein-Langevin clock at the origin of the system K are in accord with the inverse Lorentz transformation.

These time and distance values, when the Einstein-Langevin clock is treated to be at rest and system K to be in motion, are the same as those when the Elnstein-Langevin clock was considered to be in motion and the system K was considered to be at rest.

Time on the Einstein-Langerin clock is less even though it remained stationary.

As the velocity of the ray of light has been treated as C in the system K' time on the Einstein-Langevin clock will be considered to have run uniformly, whereas in the system K, it will be considered to have run non-uniformly, three times faster than the first K' second and three times slower than the next K'-second.

(b) We now suppose that the rod AB 5 K-legs long of the Einstein-Langevin clock is placed in the system K which is treated as at rest and the system Ḱ to be in motion towards the right, i.e. towards the positive side of the x axis at 4 K-legs per one K-second.

The ray of light will take one K-second to travel from the end A to the end B and one more K-second to travel back from the end B to the end A of the rod so it will take two K-seconds for its two way travel to arrive hack at the end A.

As judged from the system K, the ray of light advances towards the right at 5 k-legs per one K-second and the system K' moves forward at 4 K-legs per one K-second. The velocity of the ray of light in the system K' will thus be c-v and it will gain on the system K', a distance of [5-4] one K-leg in one K-second. But in one K-second the ray arrives at the end B of the rod. Due to the motion of the system K', the length in this system is shortened by the factor 3/5 and one K-leg measures the same distance as $\left[1\times \frac{5}{3}\right]\frac{5}{3}$ K -legs. To cover the distance of ‘K' -legs, the ray of light will take $\left[\frac{5}{3}\times \frac{1}{5}\right]\frac{1}{5}$ K' -second. K' -second.

Thus, when the ray of light arrives at the end B of the rod, i.e. at a distance of 5 K-legs from the end A in the system K and opposite the end B at leg 5/3 of the system K', time in the system K is one second and in the system, K' at 5/3 K-leg, it is 1/3 K' -second.

So the coordinates in the system K of the event of arrival of the ray at the end B of the rod are

x = 5, t = 1

By means of the Lorentz transformation:

$$x' = \frac{5}{3} \quad \frac{5}{3}[5-4\times 1] or \frac{5}{3}\times 1 = \frac{5}{3}$$
$$t' = \frac{1}{3} \quad \frac{5}{3}\left[1-\frac{4}{25}\times 5\right] or \frac{5}{3}\left[1-\frac{4}{5}\right] or \frac{5}{3}\times\frac{1}{5} = \frac{1}{3}$$

Thus our calculations of 1/3 K' -second at leg 5/3 of the system K' opposite the end B of the rod in the system K, are in accord with the Lorentz transformation.

On its return journey from the end B, the ray of light will take one more K-second to arrive at the end A and time at this end will be [1 + 1] 2 K-seconds.

As judged from the system K, the ray of light will be heading towards the left at 5 K-legs per one K-second and the system K’will be advancing towards the right, i.e. towards the on-coming ray at 4 K-legs per one K-second. The velocity of the ray of light in the system K' will therefore, be c+v or [5+4] 9 K-legs per one K-second. But as length in the system K' will be shortened by the factor .due to its motion, 9 K-legs will measure the same distance as [9x5/3] 15 K'-legs. Thus, when the ray of light arrives back at the end A of the rod after one more K-second, it covers a distance of 15 K' legs backwards from leg 5/3 of the system K' and arrives opposite [5/3 -15] leg –19/3 of the system K' will, therefore, be + [1/2 + 3]19/3 K'-seconds and at the end A of the rod in the system K, it will be 2 K-seconds.

The coordinates in the system K of the event of arrival back of the ray of light at the end A of the rod, i.e. at the origin of the system K will, therefore, be

x = o, t = 2

by means of the Lorentz transformation:

$$x' = \frac{-40}{3} \quad 5/3\,[o - 2 \times 4]\,or\, 5/3 \times 8 = 40/3$$
$$t' = 10/3 \quad 5/3\left[2 - 4/25 \times 5\right] or\, 5/3 \times 2 = 10/3$$

Thus, our calculations of 10/3 K'-seconds at leg-42 of the system k; opposite the end A of the rod and 2 K-seconds at the end A of the rod of the Einstein-Langevin clocks, i.e. at the origin- of the system K are in accord with the Lorentz transformation.

Here the system K' has been considered to be in motion and the Einstein-Langevin clock in the system K to be at rest, but time in the moving system K' is greater than the time on the Einstein-Langevin clock in the system K which has been considered to be stationary.

As the velocity of the ray of light has been considered to be C in the system K, the time on the Einstein-Langevin clock in this system will be considered to have run uniformly, whereas in the system, K', it will be considered to have run non-uniformly, thrice slow for the first K-second and thrice fast for the next K-second.

In the above three analyses, the quantity of time has been found to be less on the Einstein-Langevin clock, irrespective of the fact whether this clock was thought to be at rest or in uniform motion. Accordingly, the usual statement that " a moving clock runs slow" or the alternative statement that "time itself slows down in moving systems" is not adequate to the actual situation. Certain relativists became aware of the misleading nature of the statement, but the reason which occurred to them for the error is not satisfactory. For example, Sir O.R. Frisch wrote the following in his article, "Time and Relativity"[91]

It is vague and misleading to say, "a moving clock goes slow". To be precise, one should say "a clock moving at speed v relative to an inertial frame containing synchronised clocks is found to go slow by the factor $\sqrt{1-{v^2}/{c^2}}$ when timed by those clocks.

This seems to be in line with Einstein's statement in his first paper on special relativity theory,[92] viz.

If at points A and B of K there are synchronised clocks which, viewed in the stationary system, are Synchronous; and if the clock A is moved with velocity V along the line AB to B, then on its arrival at B, the two clocks no longer synchronise, but the clock moved from A to B lags behind the other which has remained at

B……..

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[91] O.R. Frisch, "Time and Relativity" in Contemporary Physics, Oct, 1961 pages 16-27 and reprinted in Special Relativity Theory, Selected Reprints, American Institute of Physics, pages 89-100.

[92] Einstein, "Electrodynamics" reprinted in the Principle of Relativity, Du'. er Publications Inc. page 49.

In both these statements, the clock in question is considered to have moved. But it is not on account of its motion that it lagged behind in time. The real reason for less time on the Einstein-Langevin clock is the manner in which time is measured for the travel of light in the two systems.

In the above analyses, the velocity of the ray of light is considered as C in the system which is thought to be at rest, but in the system which is thought to be moving, length is treated as shortened by the factor $\sqrt{1-v^2/c^2}$ and the velocity of the same ray of light is considered to be c-v and c+v in opposite directions. As considered to be of measuring time, the ray of light travels equal distances in the one system for its outward and return journeys, i.e. in this system the Einstein-Langevin clock is used for measuring time. In the other system, this clock is not used and the same ray of light has to travel unequal distances for its two-way journeys on account of its velocity as c-v and c+v in the system which is considered to be in motion. Its total distance of travel, therefore, gets increased in the second system by the factor of our example, from the total distance of travel of the same ray of light on the Einstein-Langevin clock. The increase in distance gives rise to the increase, by the same factor, in the time of travel of the ray of light in the second system as compared to the time of travel of the same ray on the Einstein-Langevin clock. It is, therefore, highly inappropriate to say that 'a moving clock runs slow' or 'time slows down in the moving system' with the suggestion that it is its movement which causes time" retardation. This is not the case. The increase in time occurs in the system in which the ray of light does not travel equal distances for its two way journeys as compared to the time of travel in which the same ray travels equal distances for its outward and return journeys. So the Linstein-Langevin clock, even if stationary, can be considered to go slow, to use the usual relativistic jargon, and to show less time.

It must he noted that in his first paper on special relativity theory, Einstein considered a point x'in the moving system Ḱ to which a ray of light travelled equal distances for equal times outward and back. He, then proceeded to derive the Lorentz transformation, thus using in effect, the Einstein-Langevin clock in the system K' and calculating the time in the other system K by the same ray of light. The procedure of making the ray of light travel equal distances, out and back, in one system and unequal distances in the other system, however, lacks rationale and seems to be arbitrary. (To be continued)