

# TIME IN SPECIAL RELATIVITY THEORY

## Part-II

Aziz Ahmad

### 5- A MISUNDERSTANDING BY EINSTEIN AND OTHERS.

In his book, *The Meaning of Relativity* Einstein took into consideration the two events of the emission and reception of a ray of light and while deriving the Lorentz transformation, wrote the following:

Before we analyse further the conditions which define the Lorentz transformation, we shall introduce the light time,  $l = ct$ , in place of the time,  $t$ , in order that the constant  $C$  shall not enter explicitly into formulas to be developed later.

It is unnecessary to surmise why Einstein wanted to exclude the constant  $C$  from “the formulas to be developed later”. What is important to note is the fact that the constant  $C$  performs the function of life and soul of his special theory and as such cannot be ignored, and that the so-called light-time  $l = ct$  is not time, but distance travelled by light in the time  $t$  at the velocity  $C$ .

Two pages ahead Einstein wrote the first Lorentz transformation as under:

$$z' = x - vt$$

(29)

A few lines onwards, he wrote the following:

if we introduce the ordinary time  $t$ , in place of the light-time  $l$ , ‘ then in (29) we must replace  $l$  by  $ct$  and  $v$  by  $v$ .

When  $v$  is substituted by  $v$  and  $l$  is substituted by  $ct$ , the first Lorentz transformation equation written above as (29) then becomes

$$XrY .$$

XI ' wL

Einstein wrote the fourth. Lorentz transformation as under: — t - yxi  
i- v2

Substituting for the symbols 1' and 1, . the symbols ct' and ct respectively and for the symbol v, the symbol as per Einstein quoted

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above, the equation becomes

ct' ct . z~

The symbols ct'and ct and x,stand for the distance ~travelled by li In order to convert these distances into times of travel of light, need to divide them by C. The equation then becomes

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transformation equations need to be written as - x . ct

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It is now possible to direct attention to an unfortunate mistake or misunderstanding which somehow cropped up in Eintein's mind and which continues to exist in the minds of the relativists, physicists and mathematicians till today. The fundamental problem which Einstein had set up to solve was, in his own words, the following 15:

What are the values x, y; z, t; of an event with respect to K

when the magnitudes x

... of the same event with respect to K. The relation must be so chosen that the law of the transmission of light in vacuo is satisfied for one and the

same ray of light (and of course for every ray) with respect to K and K'

This problem was solved by means of the Lorentz transformation

But very unfortunately, Einstein fell victim to the initial assumption that these transformation equations will be serviceable in respect of the

time and place of any event whatsoever which occurs anywhere in the

system K. In his first paper on relativity, he wrote the following: "To any system of values  $x, y, z, t$ , which completely defines the place and time of an event in stationary system, there belongs a system of values  $x', y', z', t'$ ; determining that event relatively

to the system K' and our task is now

to find out the system of equations connecting these quantities.

By the phrase "any system of values  $x, y, z, t$ ," Einstein was definitely contemplating any event anywhere in the stationary system K. But in order to derive the Lorentz transformation equations, he had to restrict himself to the consideration of the special events of the emission and

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reception of a ray of light. As is apparent from our restructured Lorentz transformation,

$$x' = \gamma (x - vt)$$

Z

$$t' = \gamma \left( t - \frac{vx}{c^2} \right)$$

the symbol  $ct$  in the first equation denotes the distance travelled by the ray of light in the time ending with the instant  $t$  and the symbol  $x$  in the second equation stands for the time of travel of the same ray, from the point of emission to the point  $x$  where it is made to terminate in the system  $K$ . The Lorentz transformation, therefore, is applicable only in respect of the event of reception of the ray of light at the instant  $t$  at some point  $x$  in some inertial coordinate system. It is not applicable to other events occurring at places which lie beyond the point to which the ray of light can reach in the specified interval of time from the origin of the coordinate system. Its primary concern is with the place and time of the event of reception of the ray of light relative to the various inertial systems. This fact does not seem to have occurred to Einstein's mind nor to the minds of the subsequent physicists and mathematicians. It may apply secondarily, if at all, to other events only if these happen to occur at the same place and time as of the event of the reception of the ray. In other words, if in the system  $K$ , the distance of the event from the point of emission of the ray is greater than the distance which the ray of light can cover till the instant  $t$  of the occurrence of the event, the Lorentz transformation cannot enable us to find out, the place and time of the same event relative to the moving system  $K'$ ; because  $X$  of the distance  $ct$  and  $c$  of the interval of the restructured Lorentz transformation equations concern only the distance coverable by the ray of light in the interval of time ending with the instant  $t$ , which distance in the case under consideration will be less than the distance of the event from the point of emission of the ray. Thus the Lorentz transformation has nothing to do with the events occurring in the region termed "elsewhere" in the Minkowski diagram, in which the distance  $x$  of an event is greater than the distance  $ct$  which the ray of light can cover till the instant  $t$  at the velocity  $C$ . The range of applicability of the Lorentz transformation is, therefore, restricted by the light travel distance  $ct$  and/or the light

travel time From the  $x$  and  $t$  of the event of reception of the ray of light in the system  $K$ , the  $VG$  of  $ct$  and the  $tC$  of  $c$  have to be subtracted respectively in order to obtain the  $x'$  and  $t'$  of the same event of the

reception of the ray of light in the system  $K'$ ;      i.,

Minkowski Diagram

Through an unwarranted application of the Lorentz transformation to the events in the “elsewhere” region of the Minkowski diagram the peculiar nature of these events has been elicited to be such that if an event P occurs after event o in the system K, then a system K<sub>1</sub> can be specified in which the same event P occurs before event p, and also! third system K<sub>2</sub> can be found with respect to which these two events o and p occur at the same time. Probably it was this peculiarity of the events in the region termed “elsewhere” in the Minkowski diagram which prompted H. Weyl<sup>17</sup> to his celebrated remark that, “The objective world simply is, it does not happen.” Or in the words of Oliver Costa De Beauregard,<sup>18</sup> “relativity is a theory in which everything is “written” and where change is only relative to the perceptual mode of living beings.” Or in simple words, “events do not happen; we simply meet them.” The import of these jargons exhibits itself as a wonder piece in the hands of the admirers of the theory, according to whom the totality of events of the universe is given and there is no such thing as happening or occurring of events. The late Allama Iqbal could not reconcile himself with this result of the theory which conflicted with his view of time<sup>19</sup> as a “free creative movement.” But as pointed out above, the events in the “elsewhere” region of the Minkowski diagram are not within the scope of the theory and so the earlier-later temporal order of events is not abolished and stands unscathed.

By eliminating the symbol  $c$  from the terms  $yo/ct$  and making  $vt$  of them in the first Lorentz transformation equation, and by amalgamation of the terms  $x/c$  into  $vx/c$  in the second Lorentz transformation equation, the real physical intent and significance of these terms gets severely damaged and obscured so as to promote serious misconceptions. If so, there is here a little cautionary lesson for the mathematical physicists to exercise care while manipulating terms which stand for physical quantities so that the physical purport of these terms may not get lost through elimination of certain essential symbols.

## 6. A LOGICO MATHEMATICAL ERROR.

Let us calculate by means of the Lorentz transformation, the time of the system k' at the end A of the rod, i.e. at the origin o' of the system K' when the

ray of light reaches the other end B of the rod AB. The ray of light reaches the end B in one K'-second and at leg 15 of the system K opposite the end B in 3 K-seconds. In 3 K-seconds, the end A of the

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rod which moves in the system K, covers a distance of [3 x 4] 12 K-legs at the velocity of 4 K-legs per one K-second. So the coordinates of the event of arrival of the end A opposite leg 12 of the system are

$$x = 12, \quad t = 3$$

By use of the Lorentz transformation:

$$x' = \gamma (x - vt) = \frac{1}{\sqrt{1 - \frac{16}{16}}} [12 - 4 \times 3] = 0$$

$$t' = \gamma (t - \frac{vx}{c^2}) = \frac{1}{\sqrt{1 - \frac{16}{16}}} [3 - \frac{4 \times 12}{16}] = 0$$

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Thus according to the Lorentz transformation, the time at the end

A of the rod in the system K' turns out to be 0 K'-seconds instead of one K'-second.

Here are two events, one the arrival of the ray of light at leg 15 of the system K and two, the arrival of the end A of the rod opposite leg 12 of the system K. Both occur after 3 K-seconds. They are, therefore, simultaneous in the system K. But in the system K', the same ray of light arrives at the end B of the rod and opposite leg 15 of the system K in one K'-second. Therefore, when the first event occurs, time at the end B of the rod is one K'-second. In this one K'-second, the ray of light travelled 5 K'-legs starting from the end A at zero hour. -

So, when the ray covers a distance of 5 K'-legs and arrives at the end

B of the rod and opposite leg 15 of the system K, the time at the end A, as judged from the system K; need also be one K'-second. But according to

the Lorentz transformation, when the ray arrives at the end B of the rod and opposite leg 15 of the system K, time at the end A in the system K', is not one second, but  $Q/5 K'$  -seconds. In the system K, therefore, these events are not simultaneous, the one occurs after one K' -second and the other occurs according to the Lorentz transformation, after  $V$ -seconds.

Einstein is said<sup>20</sup> to have in mind the problem of measuring time since the age of sixteen years. During the course of his thought and speculation, he must have come across such knots as the above where the Lorentz transformation allots different times at the two ends of the rod AB. It is stated that

Einstein suddenly realized one morning in May 1905 that there was a great gap in the classical treatment of time and that it is not obvious that two events in different places which are simultaneous for one observer must necessarily be simultaneous for another.

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Such an intuition

complications as the above must have enabled him transformation even if they n dismiss st as mere unfamiliar facts and to opt for t

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remainder[-: is divided by the length contraction factor 5/5, it

comes out to be 1 second which is the K'time for the arrival of  
the end A of the rod at leg 12 of the system K.

If one K' -second of time at the end A is correct, then the two events, )  
i- the event of arrival of the ray of light at the end B and opposite leg 15 of  
the system, K and ii-) the event of arrival of the end A opposite leg 12 of the  
system K are simultaneous events also in the

system K°

In the restructured Lorentz transformation

$t' - t - \frac{v}{c^2} x$  ..

V

T  $V^2/C^2$

the term stands for the light travel time at the velocity  $C$ , but here we are concerned with the travel time of the end  $A$  at the velocity  $V$

4 K-legs per one K-second. Therefore, in the above equation, as per ordinary logic, the term needed to be used should be in place of. By using the modified equation'

$c$

we get  $t \sim \frac{L}{c} - \frac{vL}{c^2}$  or  $t \sim \frac{L}{c} (1 - \frac{v}{c})$

e.i. one second of  $K$  -time at the end  $A$ . Simultaneity, therefore, seems to return in this case.

## 7. RECEPTION OF A RAY OF LIGHT AND OTHER EVENTS.

The question of time at the end  $A$  of the rod may again be considered. The end  $A$  is opposite leg 12 of the system  $K$  and time here is 3 K-seconds. The Lorentz transformation is not applicable for obtaining the time of the system  $K'$  at the end  $A$ , in the absence of the event of reception of a ray of light at leg 12 of the system  $K$  after travelling for 3 K-seconds. But as the distance of 12 K-legs from the point of emission of the ray is less than the distance of 15 K-legs which the ray of light travels in 3 K-seconds, the event of the reception of a ray of light can be associated with the event of arrival of the end  $A$  opposite leg 12 of the system  $K$ . It can be supposed that the same ray which goes upto leg 15 of the system  $K$ , is splitted up into two on the way at leg [ 12 +  $\frac{1}{2}$  ] of that system, one part going forward to leg 15 of the system  $K$  and the other part which is reflected back at leg 12 of

$Vt/W, \dots, L, CW$

the system,  $K$ , returns after travelling a distance-of  $K$ -legs to stop at 12 of that system . The reflected part of the ray will have travelled total distance of  $[12 + \frac{3}{2} + \frac{3}{2}]$  15 K-legs in 3 K-seconds. We can no calculate the distance travelled by the reflected ray in the system  $K'$ . of 12 K-legs is  $\frac{48}{3}$  Subtracting  $\frac{41}{5}$  from 12, we get When this is divide by the length contraction factor  $\frac{3}{5}$ , we get  $[12 \times \frac{5}{3} - \frac{41}{5}]$  4  $K'$ -legs travelled the ray upto leg 12 of the system  $K$ . Beyond leg: 12, the ray moves for distance of  $\frac{3}{2}K$ -legs in the forward direction and the same distance  $K$ -legs in the backward

direction and arrives to terminate at leg 12 of the system K. The  $\frac{4}{5}$  of  $\frac{3}{2}$  is  $\frac{6}{5}$ - This  $\frac{6}{5}$  is to be subtracted from  $\frac{3}{2}$  for the forward moving portion of the ray and this same factor is to be added for the reflected and backward moving portion of the ray. Therefore,

from  $[\frac{3}{2} + \frac{3}{2}]$

3 K-legs for the journey of the ray beyond leg 12 and back to this leg 12 in the system K, neither the above  $\frac{6}{5}$  legs need be subtracted nor these be added to 3 K-legs for obtaining the distance of the system K in respect of these portions of the light travels. Dividing these 3 K-legs by the length contraction factor  $\frac{3}{5}$ , we get  $[\frac{3 \times 5}{3}]$  5 K'-legs. Thus, the total K' distance travelled by the reflected ray comes to  $[4 + 5]$  9 K'-legs which, when divided by 5, the velocity of light, gives the time  $\frac{9}{5} K'$  seconds, obtainable also by the use of the usual or restructured Lorentz transformation.

It may be pointed out that it is not necessary for the ray of light to get splitted up and reflected back only at leg  $\frac{27}{2}$  of the system K. This may happen at leg 12 thereof, the reflected ray travelling  $\frac{3}{2}$  legs backwards and after getting again reflected at  $[\frac{12 - \frac{3}{2}}{2}]$  K-leg, advancing forward by  $\frac{3}{2}$  legs to arrive and stop at leg 12 of the system, K. In fact, this additional two way travel of the ray by  $\frac{3}{2}$  K-legs each way, can happen anywhere before leg 12 of the system K. Nor is it necessary that there should be a single ray which should require to be splitted up on the way. Two rays or as many as needed can be supposed to be emitted at zero hour when the origins of the coordinate systems coincide, so that each ray may be suitably reflected back at some stipulated leg of the system K.

In Section 3 above, we obtained the result that in our numerical example, the ray of light travels three times less distance in the system K' than in the system K due to its velocity being treated as  $c-v$  in tie. system K' when this ray moves forward; and it travels three times more. distance in the system K' due to its velocity being treated as  $c+v$  when. it moves backwards. In the case of our reflected ray, its. distance of, forward movement is  $[\frac{12 + \frac{3}{2}}{2}]$   $\frac{27}{2}$  K-legs and its distance of backward. movement is  $\frac{3}{2}$  K-legs. For the  $\frac{27}{2}$  K.-legs, the forward distance in the

system K' will be  $\frac{27}{2} \times \frac{1}{3} = 9$  K'-legs and for  $\frac{3}{2}K$ -legs, the backward distance in the system K' will also be  $\frac{3}{2} \times 3 = \frac{9}{2}$  K'-legs. Thus, the forward distance and the backward distance travelled by the ray in the system K', will each be  $\frac{9}{2}K'$ -legs. This brings out the peculiar fact that if a ray of light is emitted at some point A and after reflection at some other point B, is received back at A, it travels equal distances both ways. So our reflected ray travels the total distance of  $[\frac{9}{2} + \frac{9}{2}] = 9$  K'-legs and the travel

time for this distance is nine divided by five,  $\frac{9}{5}K'$ -seconds.

Here the event of the reception of the ray of light and the event of the arrival of the end A of the rod, occur at the same place and at the same time; each occurs at leg 12 of the system K and at the instant  $3K$ -seconds. The Lorentz transformation is applicable primarily only in respect of the place and time of the event of the reception of the ray of light and only secondarily in respect of the other events which may happen to occur at the same place and time as the event of the reception of the ray of light.

The association of the event of the reception of a ray of light with the event of arrival of the end-A at leg 12 of the system K, demonstrates how two events [the event of the arrival of the forward going ray at leg 15 of the system K and the event of the arrival of the reflected ray at leg 12 of that system] can be simultaneous in the system K, but not simultaneous in the system K'. However, the question of simultaneity will be discussed profitably in some detail in another essay, here it is enough to point out, that the lack of simultaneity in one system arises from Einstein's stipulation to treat from the point of view of the stationary system, the propagation of light in the moving system at the velocity  $c-v$  in one direction and  $c+v$  in the opposite direction.

In our numerical example under discussion, the Lorentz transformation will apply only in respect of the events occurring at the instant  $3K$ -seconds within a sphere of empty space of radius  $15K$ -legs. The events occurring at this instant in the vast world beyond this sphere will be outside the scope of this transformation. The range of applicability of the Lorentz transformation is thus restricted by the light travel distance  $ct$  and the light travel time ..

## 8. FILLING VACUOUS SPACE WITH MATTER.

It may be contended that as the physical events occurring at the instant  $3K$ -seconds, at the surface and within the vacuous, spatial sphere of radius  $15K$ -legs of our example, are within the scope of the Lorentz transformation, this transformation can be employed in respect of these events without the formality of associating the event of the reception of a ray of light with the place and time of these events. It

may be so, but in employing the Lorentz transformation directly straightaway to these events, we will be opting for ignorance essential elements of the situation and violating the scientific imp for going to the roots of the phenomena. Anyway neither Einstein nor subsequent physicists seem to be aware of this formality which results not merely in the exactitude and depth of physical knowledge, but points also to a serious objection to the theory.

Einstein was very particular about the propagation of light in  $v$  only. While laying down the example of railway embankment and moving train, he wrote,<sup>24</sup>

we must refer the process of the propagation of light [and indeed every other process] to a rigid reference-body (coordinate system.) As such a system, let us again choose our embankment. We shall imagine the air above it to have been removed.

The removal of air is needed to make the space empty, so that light should propagate in it at the velocity  $C$ . Einstein's theory is stated have given birth to the "operational method" not only in physics, but in other fields of scientific and even philosophic investigations, employing only those quantities which arise through actual or possible physical operations. In order to localise an event, he prescribed the following physical apparatus.<sup>25</sup>

... we can imagine this reference-body supplemented laterally and in vertical direction by means of a framework of rods, so that an event which takes place anywhere can be localised with reference to this framework. Similarly we can imagine the train travelling with the velocity  $v$  to be continued across the whole of space, so that every event, no matter how far off it may be, could also be localised with respect to the second framework.

Elsewhere he imagined similar synchronised, ordinary clocks to be placed within each of such framework of rods throughout space.

Nevertheless, he committed an intellectual sin against these physical provisions of his, when he wrote,<sup>26</sup>

Without committing any fundamental error, we can disregard the fact that in reality these frameworks would continually interfere with each other, owing to the impenetrability of solid bodies.

He seems to have slipped from his imagined physical realm to the realm of pure mathematics in which these frameworks could be

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supposed not to alter the situation. But solid bodies do interfere in the real world and alter the situation which had become merely imaginary and not maintainable as physically operational.

A very large number of physically interesting events may be imagined to take place at the instant  $3K$ -seconds in our sphere of empty space of radius  $15K$ -legs. To associate the event of the reception of a ray of light with each of such events and to calculate the time and place of each from the stationary to the moving system or vice versa, we will have to imagine an immense number of rays of light or photons emitted at zero hour and a matching number of material reflectors planted everywhere in the sphere to reflect the imagined rays of light to the imagined places of the events.

The objection is whether it is not blatantly self-contradictory to suppose that the space enclosed by our sphere would still be imaginable as empty so that light should be imagined to propagate in it at the velocity  $C$  in view of the existence in it of such a large number of imagined clocks, frameworks of rods and the immense number of our material reflectors spread in it throughout.

Anyway, as the Lorentz transformation is basically applicable only in respect of the events of emission and reception of the rays of light, the matter for urgent consideration by physicists and mathematicians is to examine whether the event of the reception of a ray of light is associable with

each of the momentous results of the theory, such as the law of the composition of velocities, increase in mass of the moving bodies and the relation of the proportionality of mass and energy etc. In case, the events of the emission and reception of a ray of light cannot be supposed to be associated with any result, such a result will be an absolutely invalid deduction from the Lorentz transformation.

### 9.1 TIME RUNS AT THE SAME RATE.

I suppose that the rod AB, this time  $5/2$  legs long of the Einstein-Langevin light clock is placed in the system  $K'$ , aligned along the positive  $x$  axis of this system with the end A at the origin and the end B pointing towards the positive side of the axis of  $x$ . I further suppose that another rod  $CD$   $5/2$  legs long of another similar Einstein-Langevin light clock is placed in the other system  $K$ . aligned along the negative  $x$  axis of this system with the end C at the origin and the end D pointing towards the negative side of the  $x$  axis. When the origins of the two systems coincide, a ray of light is emitted in the system  $K'$  from the end A to the end B where it is immediately reflected back and returns to the end A. Also at the instant of the coincidence of the origins of the

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two systems, another ray of light is emitted in the system  $K$  from the end C to the end D where it is immediately reflected back and returns to the end C. Here are two Einstein-Langevin clocks placed in two separate systems, working with two separate rays of light. As before the relative velocity of the two systems is 4 legs per second and the velocity of each ray of light is 5 legs per second.

When the ray of light returns to the end A in the systems  $K$ , it would have travelled a distance of  $[5/2+5/2]$  5 legs or 186000 miles and time at this end A will be  $5/5$  one second.

I ask what sort of time is this one second at the end A?

Light travelled out and back a total distance of  $[5/2+ 5/2]$  5 legs or 186000 miles. The distance is measurable by the standard metre-stick. Obviously, this one second of time will be no different from the second of



time recorded on your or mine or Einstein's wrist watch. This is the second for which the velocity of light is measured to be C per second.

At the end of this one second, the origin of the system K will be 4 legs away towards the left, because the relative velocity of 4 legs per second between the two systems is measurable in terms of this second. Accordingly, the distance between the origins of the two systems will be 4 legs, unshortened, at the end of this second of time.

In the same manner, when the other ray of light of the Einstein-Langevin clock in the other system K, returns to the end c, time at this end will be one second. This one second will also be no different from the seconds recorded on ordinary clocks and therefore, of the same significance and meaning as the second on the Einstein-Langevin clock in the other system K'. The origin of the system K' at the end of this second will be 4 legs away towards the right and the distance between the origins of the two systems will be 4 legs, unshortened as before.

After another trip by each ray of light on the Einstein-Langevin clock in its own system, the time in each system will be 2 seconds and the distance between the origins of the two systems will be 8 legs.

Time, plainly and evidently, is running at the same rate in both the systems.

It is a different matter, however, when Einstein's stipulation is followed and time in the two systems is calculated by "one and the same ray of light". Thus, the ray of light ravel from the end A to the end B of the rod in the system K, a distance of  $5/2$  legs each way out and hack, travelling a total distance of 5 legs. But in the system K, the same ray travels, as compared to the system K; three times (vide Section 3)

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more distance, i.e.  $5/2 \times 3$   $15/2$  legs for its outward journey and three times less distance, i.e.  $5/2 \times 1/3$   $5/6$  legs for its inward journey, thus travelling a total distance of  $(15/2 + 5/6)$   $25/3$  legs. Time for this distance is not one second, but  $1/5$   $5/3$  seconds, somewhat larger time.

This increase in time in the system K is due to the fact that in the system K', the ray of light travels on the Einstein-Langevin clock equal distances both ways, but in the system K, the same ray of light travels unequal distances, three times more distance as compared with the system K', for its outward journey and three times less distance for its backward journey, thereby earning the peculiarity that for the first second on the Einstein-Langevin clock in the system K', it records  $1\frac{1}{2}$  seconds

$\frac{3}{2}$  seconds i.e., three times more time, and for the next  $\frac{1}{2}$  second of that system, it records  $[\frac{1}{2} + \frac{1}{3}] \frac{1}{6}$  second, i.e. three times less time, total time thereof being  $[\frac{3}{2} + \frac{1}{6}] \frac{5}{3}$  seconds.

If there is another inertial system with relative velocity of  $\frac{7}{5}$  legs per second with respect to the system K', the time in the system K' will remain the same one second, but in the new system, the time of travel of the same ray will get increased to  $\frac{25}{24}$  seconds [the matter of calculations leading to this figure is left to the inquisitive reader himself.] In another inertial system, with relative velocity of 3 legs per seconds, the time of travel of the same ray will be increased to  $\frac{5}{4}$  second. In yet another inertial system, with relative velocity of  $\frac{24}{5}$  legs per second, the time of travel of the same ray of light will get increased to  $\frac{25}{7}$  seconds and so on in the case of other inertial systems, in relative motion with other velocities with respect to the system K'. Thus, the time of travel of the ray of light on the Einstein-Langevin clock in the system K' will be the same one second as absolutely invariant, but in every other inertial system which is in relative motion with respect to the system K; the time of travel of the same ray will get increased proportionately to the value of the relative velocity.

The same applies mutatis mutandis in respect of the second of time recorded by the other ray of light of the other Einstein-Langevin clock in the other system K.

If we term the time of travel of the ray of light for equal distances to and fro on the Einstein-Langevin clock in one system as the normal time of that system, and the time of travel of the same ray of light fro unequal distances both ways, out and back, in all other inertial systems as the conventional time, the conventional time in each case will be greater in magnitude than the normal time.

Thus, the normal time of travel of the ray of light both ways on the

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Einstein-Langevin clock in the system  $K'$ , will be one second and the conventional time measured by means of this very ray in the system  $K$  will be  $5/3$  seconds. Conversely, the normal time of travel of the other ray, both ways, on the Einstein-Langevin clock in the system  $K$ , will be one second and the conventional time measured by means of this second ray of light in the system  $K$ , will be  $5/3$  seconds.

Time will be running the same symmetrical way in both the cases of normal and conventional times in the two systems  $K$  and  $K'$  and as - such will be passing on quantitatively at the same rates.

In the case of conventional time, it seems that there is a case of speeding of time [if time can ever speed up or slow down] rather than its slowing down in the case of normal time when it is calculated by means of "one and the same ray of light" which travels in one particular system equal distances to and fro, but has to travel unequal distances in all other inertial systems on account of length contraction in the moving systems in which the velocity of the same ray of light is reckoned as  $c-v$  in one direction and  $c+v$  in the opposite direction.

But when separate rays of light which travel equal distances to and fro on separate Einstein-Langevin clocks in separate inertial systems are taken into consideration, the normal times as well as the conventional times run symmetrically in the same way, have quantitatively the same separate rates of normal and conventional times.

## 9.2 TIME RUNS AT THE SAME RATE.

As before, we suppose the two inertial systems  $K$  and  $K'$  in relative motion at the velocity of 4 legs per second, the velocity of light being 5 legs per second. The system  $K$  is considered as stationary and the system  $K'$  is in uniform motion towards the positive side of the  $x$  axis, When the origins of the two systems coincide, a ray of light is emitted towards the positive side of the  $x$  axis. When the ray reaches leg  $15/4$  of the system  $K$ , it is immediately reflected back and after travelling a distance of  $5/4$  legs, returns to leg  $15/4$ -

$5/4]5/2$  of the system K. It, therefore, travels a total distance of  $[15/4+5/4]$  5 legs in this system. Time of the system K for this distance is  $5/51$  second.

According to our numerical example, the distance travelled by the ray in the system K' [vide section 3 above] is three times less than in the system K for the outward travel of the ray and three times more for its backward travel. Therefore, in the system K1, the ray of light travels  $[15/4x$

$1/3]$   $5/4$ legs outwards and  $[5/4x 3 ]15/4$ legs backwards, travelling a total distance of  $[5/4+ 15/4]$  5K'-legs. Time of the system K' for this distance is  $[ 5/5] 1K'$

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-second.

In the system K', the ray of light travelled from the origin  $5/4$ legs forward and getting reflected there, travelled  $15/4$ legs backwards, thus reaching leg  $[5/4-15/4]$  of the system K'. But in the system K, the same ray travelled  $5/4$ legs in the forward direction and  $5/4$ legs in the backward direction, thus arriving at leg  $[15/4- 5/4]5/2$ of this system. The coordinates of the event of arrival of the ray at leg  $5/2$ of the system K in one second are:

$$x=5/2, t = 1$$

From these by means of the Lorentz transformation, the coordinates of the same event in the system K' are:-

$$x' = -5/2 \quad 5/3 [5/2 - 4x1] \text{ or } [5/3 x - 3/2] \text{ or } - 5/2$$

$$t' = 1 \quad 5/3[1 - 4/25X5/2] \text{ or } 5/3 [1 - 2/5] 5/3X3/5 = 1$$

Thus the distance and time values of the journey of the same ray of light calculated above in the numerical example for both the systems K and K' are in accord with the Lorentz transformation.

If the time of travel of the ray of light for  $[15/4, x 1/5]$  second in the forward direction and for  $[5/4x 1/5]1/4$ second in the backward direction in the system K is considered to be running at uniform pace, then from the standpoint of this system, the time in the system K' will be running three

times  $[3/4 \times 1/3] = 1/4$  or  $[5/4 \times 1/5 = 1/4]$  slower in the forward direction and three times  $[1/4 \times 3 = 3/4]$  or  $[15/4 + 1/5]$  faster in the backward direction than in the system K. Conversely, if the time of travel of the same ray is considered to be running uniformly in the system K' then from the standpoint of this system, it will be running in the system K, three times faster in the forward direction and three times slower in the backward direction than in the system K'. But for the total journey of the ray of light for 5 legs in each system, the time of travel of the same ray will be 1 second in each system K and K' and as such will have run quantitatively at the same rate.

If from leg 5/2 of the system K, the ray of light gets immediately reflected in the forward direction and travels 15/4 legs for 3/4 seconds to  $[5/2 + 15/4]$

] leg 25/4, where it gets reflected back and travels 5/4 legs for 1/4 second to  $[5/2 + 15/4 - 5/4]$

‘ ] - 20/4 or leg 5 of this system, it would have travelled another  $[15/4 + 5/4]$  5 K-legs in another one second.

In the system K', the distance travelled by the same ray of light will be  $[15/4 \times 1/3] 5/4$  K legs in the forward direction and  $[5/4 \times 3] 15/4$  K'-legs in the backward direction, the total distance travelled being  $[15/4 + 15/4]$  5 K'-legs. The time of travel of the ray of light for this distance will be 5/5 one K'-

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second. In this system, the ray of light travelled 5/4 K'-legs forward from - 5/2 K'-legs and reached  $[-5/2 + 5/4] - 5/4$  leg of this system from where it is reflected backwards and travelling 15/4 K'-legs arrived at leg  $[-5/4 + -15/4] - 5$  of this system.

Total time of travel of the ray of light since its emission from the point of coincidence of the two systems K and K' is  $[1 + 1]$  2 seconds in each system,.

After travelling for 2 seconds in each system, the tip of the ray will have arrived at leg 5 of the systems K and at leg -5 of the system K

The coordinates of the event of arrival of the ray of light at leg 5 of the system K will, therefore, be

$$x = 5, t = 2$$

From these by means of the Lorentz transformation, the coordinates of the same event in the system K' are:

$$x' = -5 \frac{5}{3} [5 - 4x^2] \text{ or } [5/3x - 3 \text{ or } -5$$

$$t' = 2 \frac{5}{3} [2 - 4/25x^5] \text{ or } 5/4 [50 - 20/20] \text{ or } 5/3 \times 30 \times 25 = 2$$

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Thus the distance and time values of the journey of the same ray of light in both the systems K and K', calculated above in our numerical example, are in accord with the Lorentz transformation.

If the ray of light continues its journey, travelling each time the additional distance of 5 legs in each system in the above manner, the time of its travel will go on increasing in each system K and K' at the rate of one second per 5 legs and as such will be running at the same rate quantitatively in both the systems.

For time to elapse quantitatively at different rates in the two systems, the required condition is that the ray of light should travel equal distances out from the point of emission and back to it, in one or the two systems in relative motion. But this is not a necessary condition for the measurement of time, because while calculating the time by means of the same ray in the other system, such as K, this condition has to be dispensed with. In the present section, we have tried to do without this condition altogether and have thereby shown that time can run quantitatively at the same rate in the two systems! which are in uniform, relative motion even though it is measured in both the systems by means of "one and the same ray of light" under the condition of the length contraction in the moving system and the velocity of light getting  $c-v$  and  $c+v$  in such a system.

This proves that it is not the clock or the time which slow

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down, but it is the particular manner in which time is calculated that this effect seems to come about.

## EPILOGUE

The aim of this essay has been a modest one of critical examination of the time concept of the special theory of relativity. If the critical examination is successful, it may be taken as an attempt at the *reductio ad absurdum* of this concept. During the course of examination, however, it transpired that the Lorentz transformation is concerned fundamentally and primarily with the events of the emission and reception of a ray of light from which it has arisen and not with any and every event whatsoever. If so, a problem arises for consideration by the physicists and mathematicians whether some of the momentous results of the theory, such as the law of the composition of velocities, increase in mass in the moving bodies, the result of the proportionality of mass and energy, etc. which are deduced by means of the Lorentz transformation, still remain sustainable in view of the above finding of the essay as well as the criticism of the time concept levelled in it.

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